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Annex No. 11 to the MU Directive on Habilitation Procedures and Professor Appointment Procedures

HABILITATION THESIS REVIEWER'S REPORT

Masaryk University	
Faculty	Faculty of Science
Procedure field	Mathematics - Geometry
Applicant	Yaroslav Bazaykin, Ph.D., D.Sci.
Applicant's home unit, institution	University of Hradec Kralove, Faculty of Science, Department of Mathematics
Habilitation thesis	Noncompact Riemannian manifolds with special holonomy
Reviewer	Yury Nikonorov, Ph.D., D.Sci., Prof.
Reviewer's home unit, institution	the Russian Academy of Sciences, the Vladikavkaz Scientific Center, Southern Mathematical Institute

Report Text (as large as the reader deems necessary) cf. attached letter

Reviewer's questions for the habilitation thesis defence (number of questions up to the reviewer)

1. Do you expect to find other interesting constructions of resolution of the standard cone over some Riemannian manifolds – apart from those considered in the thesis – that will be imply the existence of new Riemannian metrics with special holonomy groups?

Conclusion

The habilitation thesis entitled “Noncompact Riemannian manifolds with special holonomy” by Yaroslav Bazaykin **fulfils** requirements expected of a habilitation thesis in the field of Mathematics - Geometry.

Date: 1th of October 2020 Signature: Yury Nikonorov

ЮФМИ

ЮЖНЫЙ МАТЕМАТИЧЕСКИЙ ИНСТИТУТ –
филиал Федерального государственного бюджетного учреждения науки
Федерального научного центра «Владикавказский научный центр
Российской академии наук»

362025, Республика Северная Осетия-Алания, г. Владикавказ, ул. Ватутина, 53, http://smath.ru/
тел. (8672) 539861 / 547532, e-mail: backoffice@smath.ru

Report on the Habilitation thesis "Noncompact Riemannian manifolds with special holonomy" by Ph.D., D.Sci. Yaroslav Bazaykin

The thesis under consideration is devoted to the study of the geometry and topology of non-compact Riemannian manifolds with special holonomy groups.

The Habilitation thesis consists of Introduction, References, and 6 author's papers published in 2007–2011 in well-known mathematical journals, which emphasizes the quality of this study.

In Introduction, the author gives a brief review of Riemannian holonomy theory, in particular, the Berger list of holonomy groups of complete simply connected irreducible Riemannian manifolds.

Two of the most interesting cases in this list are special holonomy groups $Spin(7)$ and G_2. Only in 1989, Bryant and Salamon constructed the first examples of complete Riemannian metrics with exceptional holonomy on non-compact spaces. And only in 1996, Joyce proved the existence of corresponding compact examples. At the moment, the question of existing of Riemannian metrics with the holonomy groups $Spin(7)$ and G_2 on prescribed manifold (compact or noncompact) remains unclear.

After brief review of Riemannian holonomy theory, the author considers a special construction of resolution of the standard cone over 3-Sasakian manifold. In should be noted that the cone over a given 3-Sasakian 7-dimensional manifold M is hyperkähler, i. e. it have the holonomy group $Sp(2) \subset Spin(7)$. Fortunately, it is possible to deform the cone metric in order to resolve the singularity at the vertex of the cone and obtain a metric whose holonomy group remains in $Spin(7)$. The main technical tools in this case is the study of suitable dynamical systems, that describe important properties of the deformed metrics.

This idea was applied to obtain the main results of Papers A, B, and C. In Paper A, the author constructed some complete $Spin(7)$-holonomy Riemannian metrics on the noncompact orbifolds that are $\mathbb{R}^4$-bundles with an arbitrary 3-Sasakian spherical fiber M. He proved the existence of smooth metrics for $M = S^7$ and $M = SU(3)/U(1)$. This results were completed with results of Paper B and Paper C, where a very detailed analysis of the properties of auxiliary dynamical systems was carried out. This lead, in particular, to the construction of explicit Riemannian metrics with holonomy group $SU(4)$ in Paper C.

In Paper D, similar technical tools allowed to prove the existence of complete noncompact Riemannian metrics with G_2-holonomy on noncompact orbifolds that are $\mathbb{R}^3$-bundles with the twistor space $\mathcal{Z}$ as a spherical fiber.

All metrics constructed in Papers A – D have the cohomogeneity 1. Contrasting with this, in Paper E, the author constructed a family of four-dimensional smooth Ricci-flat Riemannian orbifolds of cohomogeneity two which possess the structure of complex line bundles.

Finally, in Paper F, the author constructed metrics with the holonomy group $SU(2)$ on the tangent bundles of weighted complex projective lines. Moreover, a geometric description of a neighborhood of the moduli space of special Kähler metrics on a $K3$-surface is given.

All results of the thesis supplied with clear and rigorous proofs. The author have used both standard tools from the differential geometry and dynamical systems, and some specially developed methods for the study of deformations of given Riemannian metrics. We may conclude that: (i) the thesis under consideration is a substantial and original contribution to Riemannian geometry, (ii) the candidate has demonstrated a good knowledge of the relevant literature and researches, (iii) the candidate has a satisfactory knowledge of the methodological techniques, (iv) the format and literary presentation of the thesis are satisfactory.

Despite the fact that the thesis is well written, several misprints could be found in Abstract and Introduction (say, "Sin(7)-holonomy" should be "Spin(7)-holonomy" in Abstract).

In any case, the above small disadvantages are not going to affect my overall positive judgement. My overall view of the thesis is that it is substantial and well-presented.

The contributions of Ph.D., D.Sci. Yaroslav Bazaykin to the theory of non-compact Riemannian manifolds with special holonomy groups were important mile stones in the development of the field.

In summary, I strongly recommend to accept this thesis for the habilitation of Ph.D., D.Sci. Yaroslav Bazaykin.

01.10.2020 Prof., Ph.D., D.Sci. Yury Nikonorov

Southern Mathematical Institute
of the Vladikavkaz Scientific Center
of the Russian Academy of Sciences,
Vladikavkaz, Markus str., 22, 362027,
Russian Federation
E-mail address: nikonorov2006@mail.ru