COMMENTARY

This thesis is dedicated to the study of the geometry and topology of non-compact Riemannian manifolds with special holonomy groups. The aim of the thesis is to develop geometrical methods of construction of noncompact Riemannian cohomogeneity one manifolds with special holonomy via deformation of standard cone metrics. The main approach considered is an analysis of non-linear ODEs which describe special holonomy reduction. The following results are obtained in the thesis: new Riemannian Spin(7)-holonomy metrics are constructed on orbifolds fibered over twistor space and quaternion-Kähler space of 7-dimensional 3-Sasakian manifolds; continuous family of Spin(7)-holonomy complete regular Riemannian metrics on R2-bundles over SU(3)/S1 was found; Riemannian G2-holonomy metrics are constructed on orbifolds fibered over twistor space of 7-dimensional 3-Sasakian manifolds; continuous family of SU(4)-holonomy metrics connecting Calabi metrics with holonomy groups SU(4) and Sp(2) was constructed; new Calabi-Yau metrics on tangent space to weighted complex projective line was constructed; the local structure of moduli space of SU(2)-holonomy metrics on K3 surface in the neighborhood of orbifold T4/Z3 was described.

Comments to Studies included in Habilitation Thesis

Study A. The construction of Spin(7)-holonomy Riemannian metrics on smooth resolutions of standard cones over the 3-Sasakian 7-manifolds diffeomorphic to R2-bundles over the twistor space was considered. System of non-linear ODE guaranteeing Spin(7)-holonomy was obtained and investigated by analytical methods. The existence of the smooth metrics for M = S7 and M = SU(3)/U(1) which were found earlier only numerically was proved. The author completed 100 % of all work related to the paper.

Study B. The construction of Spin(7)-holonomy Riemannian metrics on smooth resolutions of standard cones over the 3-Sasakian 7-manifolds diffeomorphic to R4-bundles over the quaternion-Kaehler space was considered. System of non-linear ODE guaranteeing Spin(7)-holonomy was obtained and investigated completely by analytical methods. It was shown that there are no new Spin(7)-holonomy metrics in considered class. The author completed 100 % of all work related to the paper.

Study C. A system of non-linear differential equations with 5 unknowns is fully investigated; this system is equivalent to the existence of a parallel Spin(7)-structure on a cone over a 3-Sasakian manifold. Solutions of the system corresponding to Spin(7)-structure were investigated completely by analytical methods. A continuous one-parameter family of solutions to this system is explicitly constructed; it corresponds to metrics with a special holonomy group, SU(4), which generalize Calabi's metrics. The author conceptualized the study, wrote 50 % of the first draft of the paper and led work on paper revisions.

Study D. The construction of G2-holonomy Riemannian metrics on smooth resolutions of standard cones over the twistor spaces of 3-Sasakian manifolds was considered. The metrics with holonomy groups G2 on orbifolds were found and investigated. The technique of bi-quotients of Lie groups was used for explicit describing of topology of constructed G2-orbifolds for the case of 3-Sasakian space SU(3)/U(1). The author conceptualized the study, wrote 50 % of the first draft of the paper and led work on paper revisions.

Study E. The general class of 4-dimensional Riemannian metrics of cohomogeneity two was considered. The new solution of Einstein equation with zero cosmological constant was found. Solution depends of parameter which controls the topology of underlying orbifold carrying Ricci-flat metric. The author completed 100 % of all work related to the paper.

Study F. Previously found Ricci-flat orbifold depending of topological parameter was studied. It was proved that the orbifold is diffeomorphic to co-tangent bundle on weighted complex projective line and has holonomy SU(2). Using Joyce's analytical technique the constructed metrics were used for describing the moduli space of SU(2)-holonomy metrics on K3-srface in the neighborhood of T4/Z_3. The author completed 100 % of all work related to the paper.